# APPLICATION OF THE EULERIAN-LAGRANGIAN METHOD TO WATER-JET COOLING OF A HOT MOVING STRIP

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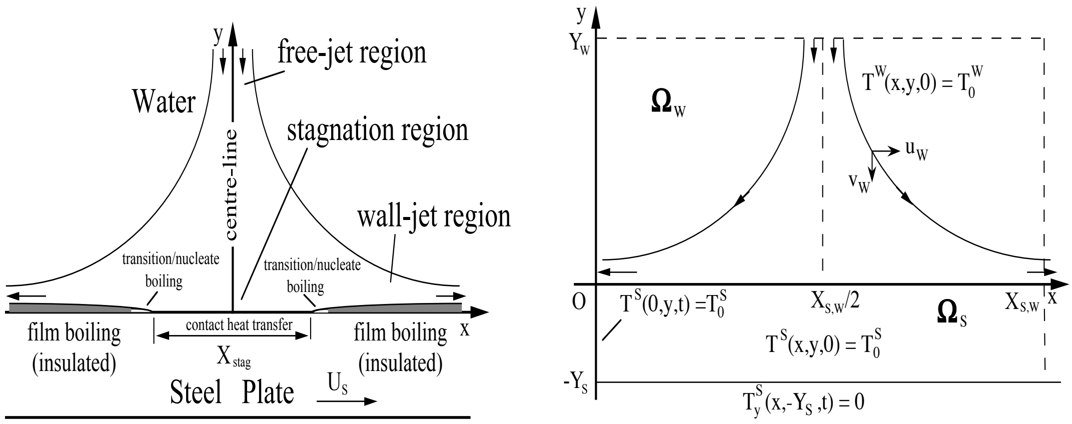
ABSTRACT. This paper investigates the heat transfer from a hot moving steel strip impinged by a laminar water-jet in the vicinity of the stagnation point with simulated film-boiling regions on either side. A two-dimensional Eulerian-Lagrangian method, in conjunction with a semi-analytical approach for large temperature gradients, has been shown to successfully solve the heat conduction equations with both advective and diffusive components. The study shows that under the impingement of the water jet the strip temperature at the stagnation point is initially depressed and subsequently rebounds strongly towards a steady state. The heat extracted by the water jet increases with increasing convective acceleration of the jet.

## INTRODUCTION

Ever-increasing demand from the automotive and building industries on sheet steel quality require steel strip to be tough, ductile, fatigue resistant, weldable and corrosion resistant [1]. These properties are achieved, in addition to other initiatives, by tight control of the finishing and coiling temperatures [2], the latter of which is achieved by precise control of the cooling at the run-out-table (ROT). Design of the ROT is governed by the considerations of high cooling efficiency and achieving the desired steel properties. Planar jets (water curtains) provide very high cooling efficiency with minimum splashing [4], but they produce non-uniform cooling on the top and bottom surface as well as over the length of the cooling zone. Spray cooling has a low specific cooling performance, and incurs high maintenance costs [1]. A compromise between high specific cooling performance and uniform strip cooling is provided by an array of round laminar jets impinging on the steel sheet. Although limited experimental studies of round jet arrays have been conducted [1], the heat transfer characteristics of single laminar jets impinging on a hot moving sheet have not yet been fully investigated.

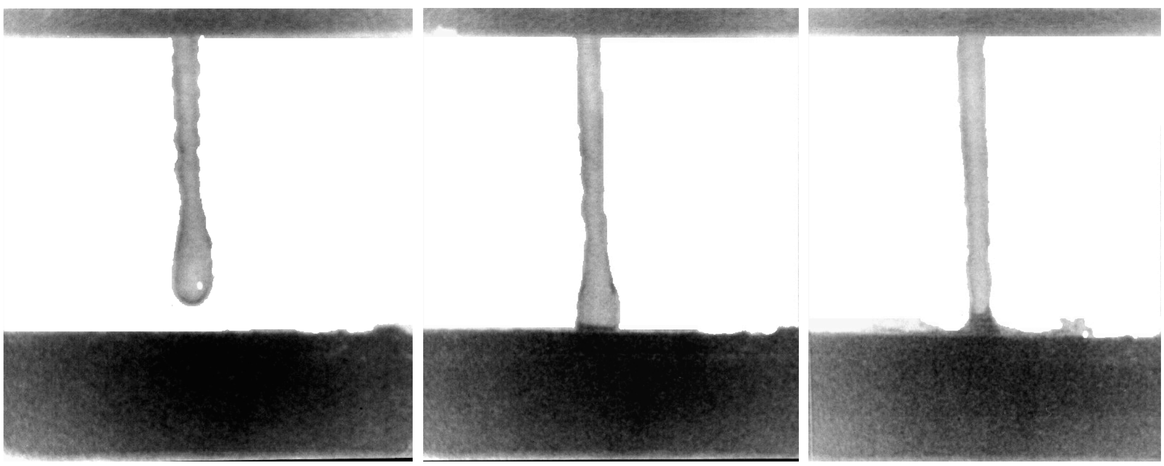
Cooling of the strip, typically in excess of $800^oC$, leads to boiling heat transfer characterised by forced convection, nucleate boiling, transition boiling and film-boiling regimes. In stationary strip jet-impingement (non-moving sheet), these boiling regimes are present simultaneously at differing distances as measured from the jet impingement centre-line, see Figure 1(a). The thermal zones for jet impingement may be delineated into a *free-jet region*, where the velocity and temperature distributions of the jet are not affected by the presence of the strip surface; a jet-impingement or *stagnation region*, where single-phase forced convection takes place over several jet widths and the cooling effectiveness is high [1,4]; and the *wall-jet region*, separated into a small region of transition and nucleate boiling before entering the parallel film-boiling region separating the strip surface from the jet by a vapour layer which reduces cooling effectiveness considerably. This region may span fifty jet widths from the jet centre-line [1] with the velocity becoming distance dependent.




(a) (b)

Figure 1. Thermal zones (a) and (b) computational domain of the water-jet cooling of a hot moving strip.

Although the strip surface temperatures in the stagnation region exceed the minimum film-boiling temperature it has been shown that the jet momentum and a high degree of water-jet sub-cooling inhibits the formation of a vapour layer, thus allowing wetting of the strip surface [4]. The darkening of the red-hot surface (steel at $1000^oC$) in the impingement region, which only occurs below $500^oC$, confirms this assumption [3]. In addition, high measured heat transfer rates [5] and high speed photography of jet impact on a hot surface (Figure 2) show direct contact (see Soh and Yuen [2]) and a suppression of both the thermal and hydrodynamic boundary layers within one jet width of the stagnation region [6]. Consequently, higher jet impact velocities are expected to intensify local heat transfer [6].



(a) (b) (c)

Figure 2. Photographs showing contact between a water-jet striking a plate initially heated to $900^oC$: (a) just before the impact, (b) 2 ms and (c) 5 ms after impact of the jet. No boiling was observed in this duration.

For moving strip impingement boiling, viscous forces will elongate the impingement zone in the direction of motion thereby enhancing heat transfer downstream of the stagnation point (SP) and reducing it upstream [1]. Computations have shown that the cross-sectional strip isotherms are also stretched downstream [2]. If the strip speed exceeds that of the impacting fluid, it significantly influences heat transfer in the film-boiling regime even in the case of high water sub-cooling [4]. However, in order to simplify the calculations in this paper, and in line with previous work [2], it will be assumed that the hydrodynamic boundary layer around the SP is extremely thin.

Due to both fluid flow and strip movement, the water-jet impingement cooling of a hot strip involves the solution of transport equations under certain initial and boundary/interface conditions. Limitations in the analytical solutions of such equations demand a numerical approach. Traditional Eulerian finite difference methods for the solution of such advection equations incur severe Courant number stability restrictions and Peclet number induced spurious oscillations [8] which are not improved by upwinding methods (e.g. Quy et al. [9]). Purely Lagrangian methods which deal admirably with advection problems are not so successful when diffusion is present. A combined hybrid technique called the Eulerian-Lagrangian method (ELM), which is the technique adopted in this paper, has been shown to be highly effective in the solution of the transport equation even for very large Peclet numbers and Courant numbers in excess of one [8].

This paper aims to investigate how the unsteady water-jet cooling of a moving plate near the SP effects the surface heat flux and internal temperature structure of the plate, taking into account the jet velocity distribution, vapour layers in the film-boiling region and heat transfer across the water/plate interface.

## THE MATHEMATICAL MODEL

Consider a water (W) jet (Figure 1(b)) with an initial temperature $T_0^W$, impinging on a hot steel (S) plate, with an initial temperature $T_0^S$ and thickness $Y_S$, moving at a constant velocity $U_S$ in the positive x-direction in the stationary frame of an xy-coordinate system defined by $\Omega = \Omega_S \cup \Omega_W$, where $\Omega_S = \{(x,y), x \in (0, X_S), y \in (-Y_S, 0)\}$ and $\Omega_W = \{(x,y), x \in (0, X_W), y \in (0, Y_W)\}$. The steel plate exits a strip-roller at $x = 0$ which is maintained at a constant temperature $T_0^S$, the underside, $y = -Y_S$, is assumed insulated. These two domains are separated by the water/steel interface at $y = 0$ with two insulated vapour layers on either side of a stagnation region of width $X_{stag}$, see Figure 1(a). Since the water impact is assumed frictionless, a steady velocity field which depends on the distance from the SP is defined by: $u_W(x,y) = \lambda(x - X_W/2)$ and $v_W(x,y) = \lambda y$, where $\lambda$ is a constant. The computational domain extends from the strip entering the jet cooling affected zone on the left at $x = 0$ up to $x = X_W$, where the RHS temperature boundary condition (BC) is extrapolated from internal values. The water-jet enters $\Omega_W$ along the line $x = X_W/2$ at a constant temperature $T_0^W$, with the temperature at $x = 0$ and $x = X_W$ extrapolated. Thus:

$$\begin{array}{ll} DT^W/Dt = \alpha_W \nabla^2 T^W & DT^S/Dt = \alpha_S \nabla^2 T^S \\ T^W(x,y,0) = T_0^W & T^S(x,y,0) = T_0^S \\ T^W(0,y,t), T^W(X_W,y,0) \text{ extrapolated} & T^S(0,y,t), T^S(X_S,y,0) \text{ extrapolated} \\ T^W(x,Y_W,t) = T_0^W & \partial T^S(x,-Y_S,t)/\partial y = 0 \end{array} \qquad (1)$$

The interface temperature distribution is calculated from temperature and flux continuity if inside the stagnation region, and insulated conditions if in the film-boiling/vapour layer region:

$$\begin{array}{ll} \left\{ \begin{array}{c} T^S(x,0,t) = T^W(x,0,t) \\ k_S \partial T^S(x,0,t)/\partial y = k_W \partial T^W(x,0,t)/\partial y \end{array} \right. & (X_{S,W} - X_{stag})/2 \le x \le (X_{S,W} + X_{stag})/2 \\ \partial T^S(x,0,t)/\partial y = \partial T^W(x,0,t)/\partial y = 0 & \begin{array}{c} 0 \le x < (X_{S,W} - X_{stag})/2, \\ (X_{S,W} + X_{stag})/2 < x \le X_{S,W} \end{array} \end{array} \qquad (2)$$

Here, the co-moving derivative indicates the time rate of change is calculated along a characteristic line defined by the solution of:

$$\dot{x} = u(x, y) \qquad \dot{y} = v(x, y) \tag{3}$$

where $D/Dt \equiv \partial/\partial t + u(x, y)\partial/\partial x + v(x, y)\partial/\partial y, u = u_W, v = v_W$ in $\Omega_W$ and $u = u_S, v = v_S$ in $\Omega_S$. The constant diffusion coefficients are defined by $\alpha_{S,W} = k_{S,W}/\rho_{S,W}c_{S,W}$ where $k_{S,W}, \rho_{S,W}$ and $c_{S,W}$ are the thermal conductivity, density and specific heat respectively.

**The Eulerian-Lagrangian method**

The solution of equations (1) subject to (2) above is carried out by the discretisation of the domain with fixed Eulerian nodes defined by $x_i = i\Delta x, y_j = j\Delta y, \Delta x = X/I, \Delta y = Y/J$ where $i, j = \{0, 1, 2, ..I; 0, 1, 2, ..J\}$ and $t^n = n\Delta t, n = \{0, 1, 2, ..\}$. Adopting the notation of Casulli [10] and since the interface conditions are defined explicitly, the 2D explicit Eulerian-Lagrangian discretisation of the transport equation is:

$$T_{ij}^{n+1} = [1 - 2(s_x + s_y)]T_{i-aj-b}^n + s_x(T_{i-a+1j-b}^n + T_{i-a-1j-b}^n) + s_y(T_{i-aj-b+1}^n + T_{i-aj-b-1}^n) \tag{4}$$

where $a = u\Delta t/\Delta x$ and $b = v\Delta t/\Delta y$ are the Courant numbers, $s_{x,y} = \alpha\Delta t/(\Delta x, \Delta y)^2$ so that $T_{i-aj-b}^n = T[(i-a)\Delta x, (j-b)\Delta y, n\Delta t]$ is the temperature back-tracked along the characteristic line passing through the point $(i\Delta x, j\Delta y)$ at time level $t^{n+1}$ and ending at the back-tracked point $[(i-a)\Delta x, (j-b)\Delta y]$ at time level $t^n$. Since this is not (generally) a grid point an interpolation formula is used to construct the value $T_{i-aj-b}^n$ which remains constant along (3). A bi-linear interpolation formula is chosen because it is stable and free from spurious oscillations provided the stability condition $\Delta t \leq [2\alpha/(1/\Delta x^2 + 1/\Delta y^2)]^{-1}$ is obeyed [9]. The interpolation formula is given by:

$$T_{i-aj-b} = (1-p)[(1-q)T_{i-nj-m} + qT_{i-nj-m-1}] + p[(1-q)T_{i-n-1j-m} + qT_{i-n-1j-m-1}]$$

where $a = n + p$ and $b = m + q$, $n$ and $m$ are the integer parts of $a$ and $b$, with $0 \leq p, q < 1$. Some artificial diffusion is present but may be reduced by decreasing the sizes of $\Delta x$ and $\Delta y$ and increasing $\Delta t$ [10]. An additional restriction needs to be imposed on the time step so that the back-tracked points do not fall too far outside the boundaries thereby reducing the need for extra ghostcells. We have chosen to confine the back-tracking to within one cell.

**Interface condition**

It is known that a stationary ($U_S = u_W = v_W = 0$) 1D version of the problem (1,2), with insulated sides, without vapour layers along the interface, and infinite $y$ boundaries, is equivalent to a one-dimensional analytic solution in $y$ with the same initial conditions in the form given by:

$$T_{1D}^{S,w} = A^{S,W} + B^{S,W} erfc\left(\pm y/2\sqrt{\alpha_{S,W}t}\right) \tag{5}$$

where $A^{S,W} = T_0^{S,W}$, $B^{S,W} = \pm k_{S,W}\sqrt{\alpha_{W,S}}(T_0^S - T_0^W)/(k_S\sqrt{\alpha_W} + k_W\sqrt{\alpha_S})$, "$+$" $\equiv y \geq 0$, "$-$" $\equiv y \leq 0$ The gradient $\partial T/\partial y$ is very large at the interface and any first order discretisation of the interface conditions (2) will give rise to large errors in this region. Instead we use a second order parabolic approximation for the derivatives in (2) such that the interface temperature at time is given by:

$$T_{i0}^{S,W} = \frac{(k_W\Delta y_S(4T_{i1}^W - T_{i2}^W) + k_S\Delta y_W(4T_{i-1}^S - T_{i-2}^S)}{3(k_S\sqrt{\alpha_W} + k_W\sqrt{\alpha_S})} \tag{6}$$

In addition, we separate the total numerical temperature field into a 1D analytic component (1D) and a perturbed numerical component (p): $T^{S,W}(x,y,t) = T^{S,W}_{1D}(y,t) + T^{S,W}_{p}(x,y,t)$. The large size of the gradient is now encapsulated in the analytic component. The interface calculation takes place in three steps, at $t^n$ after $T^n_{ij}$ has been calculated for $j = \{-J_S, .., -1\}$ and $T^W_{ij}$ for $j = \{1, .., J_W\}$ then:

- $T^S_{p_{i-1,-2}} = T^S_{i-1,-2} - T^S_{1D_{-1,-2}}, T^W_{p_{i1,2}} = T^W_{i1,2} - T^W_{1D_{1,2}}$
- use (6) to get $T^{S,W}_{p_{i0}}$
- $T^S_{i-1,-2} = T^S_{p_{i-1,-2}} + T^S_{1D_{-1,-2}}, T^W_{i1,2} = T^W_{p_{i1,2}} + T^W_{1D_{1,2}}, T^{S,W}_{i0} = T^{S,W}_{p_{i0}} + T^{S,W}_{1D_0}$

These new interface values are then used in the next time step.

## RESULTS

The following parameters were used to study the jet impact cooling process at the stagnation point (NB: due to the high temperature gradients at $y = 0$ and to maintain resolution $\Delta y_W \ll \Delta y_S$, the grid given below was the minimum required for accuracy and calculation time constraints):

Table 1
Jet and Plate Parameters

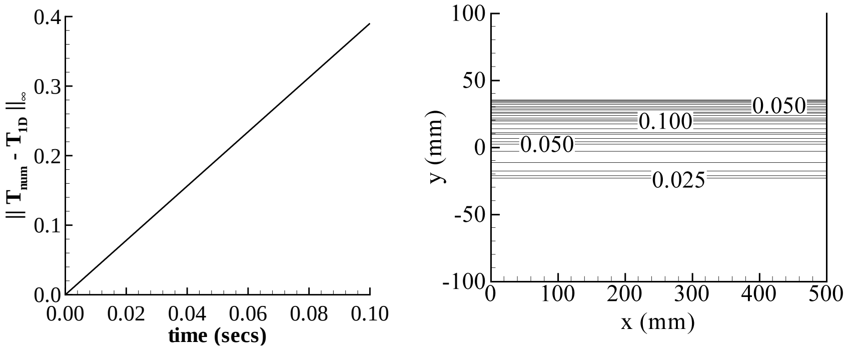
| | | | |
|---|---|---|---|
| Observation window $X_{S,W}$ | 0.5 m | Total time of calculation | 0.1 s |
| stagnation region $X_{stag}$ | 200 mm | Speed of Plate $U_S$ | 10 m/s |
| Steel Plate thickness $Y_S$ | 5 mm | Initial water temp. $T^W_0$ | $30^oC$ |
| Water Layer thickness $Y_W$ | 1 mm | Initial plate temp. $T^S_0$ | $1000^oC$ |
| $\Delta x_S = \Delta x_W = X_{S,W}/I_{S,W}$ | 10 mm | $\Delta y_S = Y_S/J_S$, $J_S = 50$ | 0.1 mm |
| $I_{S,W} = 50$ | | $\Delta y_W = Y_W/J_W$, $J_W = 50$ | 0.02 mm |

Table 2
Thermophysical Properties

| | Density $\rho$ $(kg/m^3)$ | Specific Heat $c$ $(J/kgK)$ | Thermal Conductivity $k$ $(W/mK)$ |
|---|---|---|---|
| Water | 1000 | 4200 | 0.597 |
| Steel | 7897 | 473 | 40 |

### Numerical test

The numerical solution of (1) with the same initial conditions and the first two interface conditions of (2) was tested against the 1D analytic solution (5) (no non-stationary analytic solutions are available) by imposing left and right insulation boundary conditions along with the upper and lower boundary conditions of (1) as $Y_{S,W} = \pm 1m \simeq \pm\infty$ while ensuring $U_S = u_W = v_W = 0$. The absolute and relative percentage errors were calculated over the entire domain $\Omega_S \cup \Omega_W$, see Figure 3.

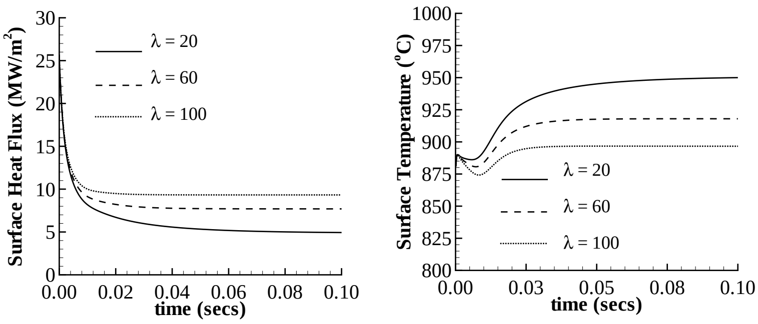



(a) (b)

Figure 3. (a) Absolute error with time and (b) Relative percentage error contour plot for the whole temperature field compared to the 1D solution $(T_{num} - T_{1D})/T_{1D}$ after 0.1 s.

### Surface heat transfer at the stagnation point

For the jet cooling case (Eqn (1) with conditions (2) and Tables 1 & 2) the graphs of the surface/interface temperature and heat flux at the SP for various velocity distributions were obtained as a function of time up to 0.1 s, after which an approximate steady state was reached. These are shown in Figure 4. Cross-sectional temperature profiles for the case $\lambda = 100$ were obtained with time, see Figures 5 and 6.



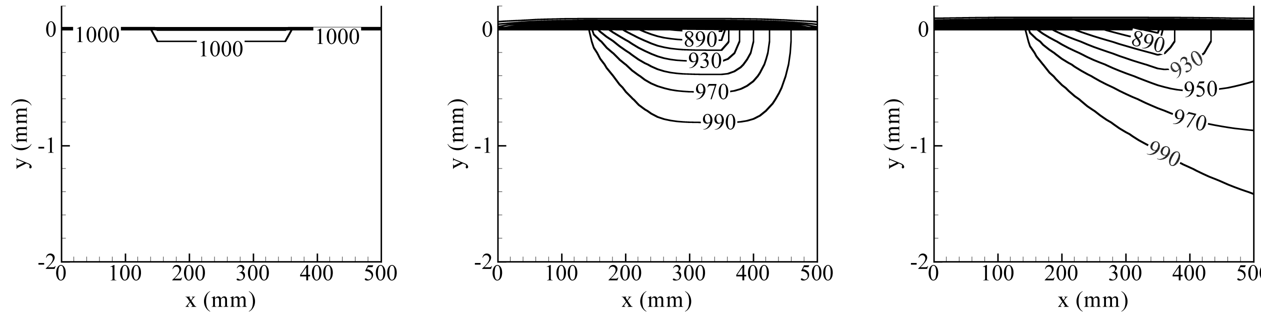

(a) (b)

Figure 4. The effect of $\lambda$ on graphs of SP surface (a) heat flux and (b) temperature versus time.

## DISCUSSION AND CONCLUSION

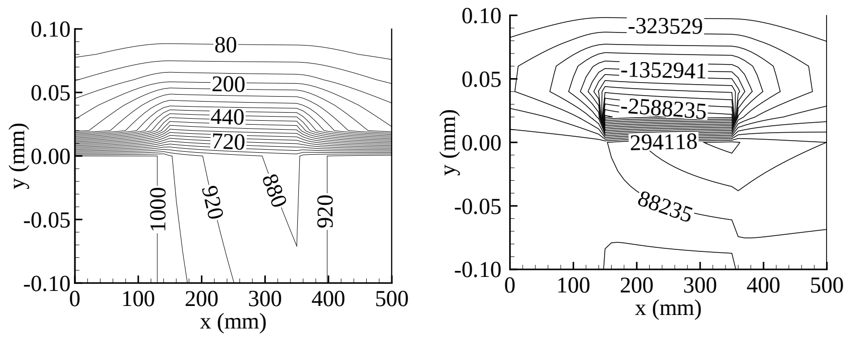
Although a linear growth in error occurs over the measured time, Figure 3(a), the maximum absolute error never exceeds $0.4^oC$. The large errors incurred in the calculation of the interface temperature can be seen in the relative percentage error contour plot of Figure 3(b). Since steady state conditions are approximated after 0.1 s and the error stays small we consider the numerical technique adequate for our present purposes.

The effect of velocity distribution on the transport of heat of a moving steel sheet impinged by a water-jet possesses two aspects, first, the transport of heat by the water velocity field momentarily decreases the surface temperature at the SP, Figure 4(b), until the much stronger transport in the steel sheet raises the temperature after a characteristic transport time scale




(a) (b) (c)

Figure 5. Cross-sectional temperature contour plots for $\lambda = 100$ in the simulated film-boiling case at (a) $t = 0s$, (b) $t = 0.02s$, (c) $t = 0.1s$.




(a) (b)

Figure 6. Detailed contour plots of (a) temperature and (b) $\partial T/\partial y$ in the vicinity of the SP after 0.1 s for $\lambda = 100$.

$X/(2U_S)$. The size of this initial temperature drop is proportional to the size of the water velocity field chosen. Secondly, Figure 4(a) shows that higher water velocities (larger $\lambda$) increase the surface heat flux which is of the same order as earlier research ($5MW/m^2$) [7]. Similarly, there is a lowering of the corresponding surface temperature at the stagnation point (Figure 4(b)) which also is in agreement with previous research [6].

Time dependent behaviour includes the development of a cooled patch, Figure 5(a)-(c), 6(a), in the stagnation region which is elongated downstream in the strip by the steel velocity transport after the characteristic transport time scale. Finally, Figure 6(b) shows the distinct size difference in $\partial T/\partial y$ above and below the interface with the gradient inside the water domain as much as 20 times greater than that in the steel, indicating the difficulty in obtaining convergent results in the water domain without using the semi-analytical treatment of the interface condition as adopted in this study.

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